The "Object Participant" Paradigm: Toward an Appropriate Use of Graphing Calculators in the Teaching and Learning of School Mathematics

Michael Todd Edwards, Miami University <edwardm2@muohio.edu>
Jamie Borchik, Miami University <camcraz1421@aol.com>

Michael Todd Edwards *is an assistant professor of mathematics education at Miami University in Oxford, Ohio. His research interests focus primarily upon teacher practice and uses of technology in the teaching and learning of school mathematics. In particular, he is interested in the impact of computer algebra systems and dynamic geometry software on classroom instruction with middle and secondary school students and their teachers.* ***Jamie Borchik*** *is an undergraduate student in the Department of Adolescent Education at Miami University. His focus of study is Mathematics and English-Language Arts Education. He coaches basketball at a local high school and directs the youth ministry at his local church.*

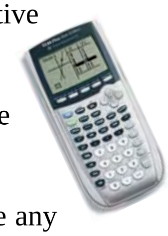
Introduction

Numerous studies have highlighted the potential of graphing calculators to transform the nature of teaching and learning in school mathematics (Gowland, 1998; Kaput, 1992; Kieran, 2005; Knuth, 2005). A growing body of research indicates that thoughtful use of graphing calculators enhances student mathematical understanding and performance (Dunham & Dick, 1994; Thompson & Senk, 2001). For instance, calculator use has been linked to higher achievement among low performing students (Harskamp, Suhre, & Van Streun, 2000), improved student knowledge of functions (Schwarz & Hershkowitz, 1999), and significant gains in concept attainment, problem solving, and operational skills (Ellington, 2003).

Despite the apparent benefits of calculator use, instructors remain slow to incorporate these tools into their classroom teaching repertoires. Often, misconceptions regarding calculator capabilities inhibit their full inclusion in mathematics classrooms (Fleener, 1995; Graber, 1993; Schmidt & Callaghan, 1992; Terranova, 1990). For instance, many believe that heavy calculator use will erode students' computational or mental mathematics skills. Others argue that calculators inhibit student mastery of basic concepts (Payne, 1996; Simonsen & Dick, 1997; Smith, 1996; Zand & Crowe, 1997). These beliefs - common among both teachers *and* students - are cultivated by *inappropriate* calculator use, particularly by inexperienced users. Throughout this article, we aim to dispel these "myths," while highlighting teaching strategies that utilize calculators as *conversation pieces* rather than *answer generation devices* (Edwards, 2001). In particular, we explore the notion of calculator as *participant* (Bernhardt, 1985) as a useful framework for considering appropriate uses of graphing calculators.

The Object / Participant Paradigm

The terms *object* and *participant*, popularized by Bernhardt (1985) as a way to discuss the interactive relationships that exist between student readers and text, are not currently part of the lexicon of mathematics teachers or mathematics educators. Nonetheless, these terms can provide the mathematics education community with a framework to discuss ideas central to the appropriate use of graphing calculators in school classrooms.



For the purpose of this essay, we define *text* as *anything* that can be read. Texts may then include any of the following: numbers, operators, activities in which the students must “read” pictures, mathematical symbols written on a chalkboard, or output on the screen of a graphing calculator. Treating the text of a graphing calculator as an *object* implies that students see the results of computations (including graphical and data-oriented displays of information) as *factual, irrefutable truth*. On the other hand, treating calculator text as *participant* implies that students are invited to see the tools' output as just *one view* of truth - often incomplete, flawed, and subject to interpretation. *Participatory text* is to be interacted with, questioned, interpreted, and connected to prior knowledge. *Objectified text* exists as content that students *must know*. Text, when treated as a participant, takes part in conversations that lead to *real knowing* (Borchik, 2006). We posit that a participatory view of the graphing calculator enables teachers and students to more fully realize technology's potential to strengthen student conceptual understanding of mathematics. The terms *object* and *participant* provide teachers with a language for discussing - and reflecting upon - the role of calculators in their teaching practice.

The Current State of Affairs

Individuals engaged in field work in mathematics classrooms soon recognize a gulf between technology recommendations set forth by professional organizations such as the *National Council of Teachers of Mathematics* (NCTM) and classroom practice of current mathematics teachers. For instance, in reference to a recent field experience in a local middle school, a teacher candidate enrolled in one of our classes made the following comments.

> I most certainly agree that calculators are objectified text in purest form. Especially after our two field observations, calculators are definitely objectified text. When students use calculators, they usually have no idea why they got that answer. However, they simply write the answer down next to the problem and move on to the next question. While doing this, they truly do not know how the calculator is *magically* getting those answers.

Preservice secondary teachers make similar observations as they observe high school students using graphing calculators.

> I was completely shocked in my last field placement. One tenth grade student actually pulled out her calculator and used the `SOLVE` command when asked to solve the equation $2x + 8 = 14$. I could not believe it. This made me realize how important it is to really teach children to understand what they are learning and made me question whether or not I even want to use calculators in my first classroom.

In our experience, comments such as these are the *norm* rather than the exception. Although we are certain that *many* teachers and students use calculators responsibly, many others do not. The observations of our preservice teachers point out a need for renewed conversation in the mathematics education community regarding *appropriate use* of graphing calculators in school.

High Stakes Testing and the Objectification of Calculator Text. Tradition, convenience, inadequate teacher training, and politics have allowed calculator text to remain an object in many of today's mathematics classrooms. In an era of high stakes testing, classroom teachers increasingly find themselves under pressure to *teach to the test* - providing students with exercises similar (if not identical) to those found on statewide proficiency tests. In a *teach to the test* environment, classroom instruction is typically teacher-centered, consisting of techniques for solving problems by means of example. In a world dominated by machine scored, multiple choice tests, *correct answers are more important than process*. Students complete exercises that encourage them to practice the mathematical routines and procedures illustrated by their teachers. Rote memorization and the execution of algorithms (the *what* of mathematics) are central to the approach. On the other hand, student *understanding* of algorithms (the *why* of mathematics) is typically *not* a central instructional consideration.

Since the *teach to the test* approach reduces mathematics problems to that of memorization and algorithmic execution, the environment encourages students and teachers to view text as object. On *high stakes* tests, answers are typically either right or wrong. When students perform an exercise such as 43 - 12 in a *teach to the test* world, they wait for an *other* (i.e. teacher, calculator, test grader) to tell them if they have answered correctly or incorrectly. The answer is 31 strictly because it is; no further explanation is needed. In the later grades, the relationship between reader and text is much the same. Students solve linear equations, perform matrix operations, calculate volumes and areas, and compute averages with little thought of what they are doing. The locus of control lies outside the student and squarely with the *other* - the teacher, the calculator, the textbook.

In this environment, we see a disturbing trend among students and teachers to relinquish ownership and control of their own learning (and their own teaching) to others. If we genuinely wish for students to "make conjectures, experiment with various approaches to solving problems, construct mathematical arguments, and respond to others' arguments" (NCTM, p. 18), then students must be encouraged to take ownership of their own learning. The subtle (and not so subtle) messages that objectified text sends to students run counter to the instructional goals of organizations such as NCTM.

Calculator Text as Participant

In the following paragraphs, we highlight several strategies we've used with school mathematics students and preservice teachers that encourage a view of graphing calculators as *participants* rather than *objects*. The approaches are highlighted through explorations of typical problems students encounter in classrooms. To help organize our research on the topic, we've organized examples of calculators as *participant* in three broad categories. These are (1) Calculator as Pattern Generator; (2) Calculator as *Confounder*; and (3) Calculator as Step-by-Step Solver. We provide an example of each type of calculator *participation* by means of short vignettes involving teaching episodes of preservice teachers.

<u>Calculator as Pattern Generator</u>

<u>Systems of Equations</u>. Lisa, a twenty-year-old college junior and preservice secondary mathematics teacher, was instructed by her cooperating teacher to lead a discussion of equation solving - more specifically, of solving systems of two linear equations Rather than introducing her lesson by telling students *how to do it*, Lisa created an exploratory lesson that used the algebraic manipulation capabilities of the *Classpad 300* graphing calculator (Casio, 2003). We describe portions of her lesson in the passages that follow.

On the morning of her first lesson, Lisa displayed the screen of her graphing calculator in front of the classroom with an overhead projector. She asked a student to type the system {$x + y = 12$; $x - y = 2$} on the calculator (a *Classpad 300*). The student entered the expressions into the calculator using templates that closely resembled the mathematical notation found in the textbook.

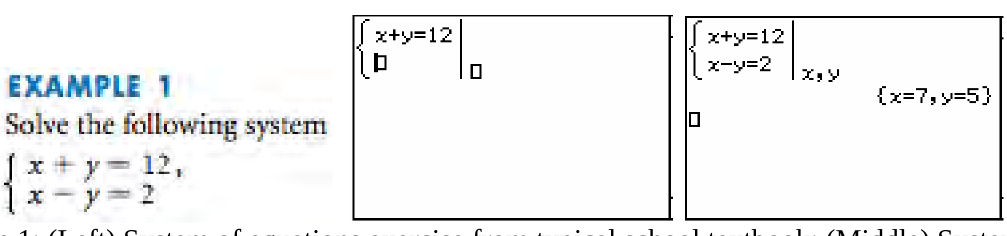


Figure 1: (Left) System of equations exercise from typical school textbook; (Middle) Systems of equations template; (Right) Results of *Classpad* system calculation.

After entering the system of equations into the calculator, the student pressed the `EXE` (i.e. execute) key. The ordered pair of values {$x = 7, y = 5$} appeared at the bottom of the screen (see Figure 1, right). A discussion ensued between Lisa and her students as they attempted to make sense of the calculator output. After seemingly endless days of lecture, the students seemed intrigued by Lisa's willingness to *discuss* mathematics with them. The calculator text provided a vehicle for such discussions.

"X equals 7? Y equals 5? What does this mean?" Lisa asked. Several students were obviously puzzled by her question, but others seemed eager to share their insights. After writing several student conjectures on the chalkboard, Lisa asked students to predict what would happen if the 12 in the first equation were replaced with a 20. "How would that change the values of *x* and *y* shown at the bottom of the screen?" she asked.

"They'll have to add up to 20 now instead of 12," offered one student.

"So maybe the numbers will be 15 and 5 instead?" Lisa asked. The room was quiet as students considered her question. After a second round of conjecturing, a student using a calculator duplicated the first system then modified the first equation and pressed `EXE`. Now the class was able to consider the effect of changing 12 to 20.

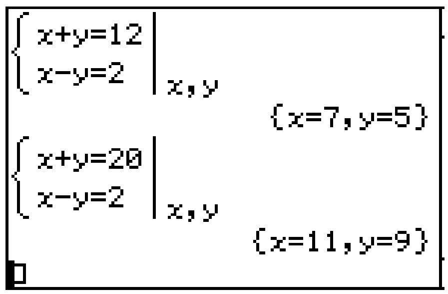


Figure 2: Two system calculations shown simultaneously on the *Classpad 300*.

"Yeah, boy! They still add up to 20," remarked a student in the back.

"And both times they were two apart," a student said flatly. "7 is 2 more than 5. 11 is 2 more than 9. See?" This student stood up and pointed to the view screen in the front of the class.

"What do you think?" Lisa asked the students. After modifying several other values in the equations, all of the students seemed to understand the relationship between the values a and b in the system $\{x + y = a; x – y = b\}$. Pointing to the original system of equations, Lisa remarked to the students that they had successfully solved several systems of equations. "The first equation says that the sum of the two numbers must be 12. And the second says the difference must be 2. Since 7 + 5 = 12 and 7 – 5 = 2, the solution to the system is $x = 7$ and $y = 5$. Good job, class!" Through their discussion, students built a solid conceptual foundation on which to build understanding of systems of equations. In particular, they began to see that a solution to a system of equations is a set of values that satisfy each equation in the system simultaneously. The previous example illustrates the pedagogical usefulness of using a graphing calculator as an *answer generator*. By examining numerous instances of a particular problem, students are encouraged to build conjectures and make meaning from the numerical relationships in calculator output.

After some discussion, students may be encouraged to look at examples that extend their initial understanding of systems of equations by linking symbolic and graphical representations. Figure 3 illustrates the results of dragging individual linear equations from a system of equations into a graphical window.

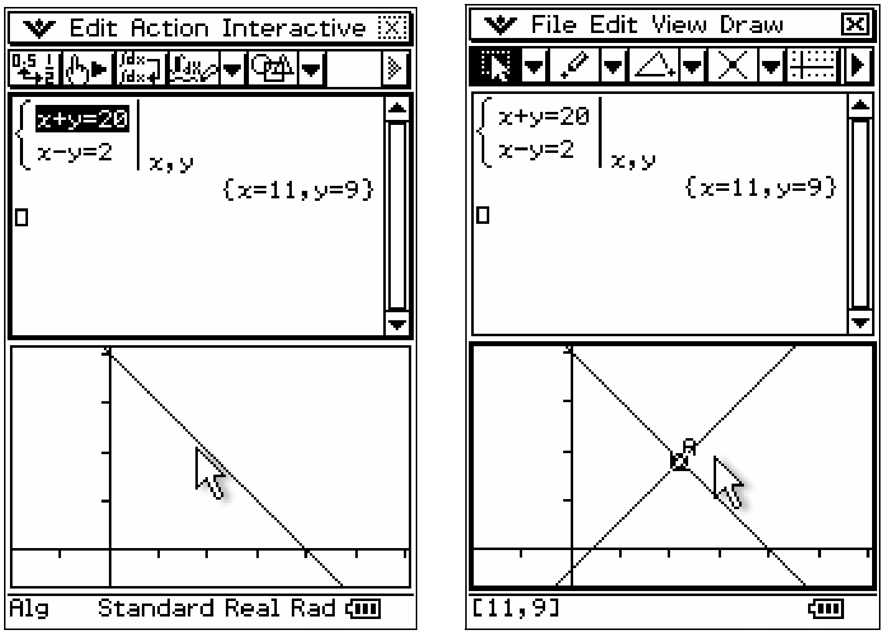


Figure 3: (Left) Dragging equation to graphical window; (Right) Connecting symbolic and graphical representations of system solution.

Using the graphing calculator, students are encouraged to see that the intersection point of the resulting lines shares the same *x* and *y* values as the symbolic solution listed in the "symbolic" window.

Calculator as Confounder

The previous example highlighted the power of graphing calculators as tools for answering student and teacher-generated questions both quickly and accurately. However, it is important to recognize that graphing calculators are also limited in what they can do. For instance, calculators have a fixed amount of memory. Most perform calculations numerically, so answers typically include round-off error. Likewise, the tools construct graphs numerically - plotting a finite number of points and connecting them to provide the *illusion* of a smooth curve. When teachers are unaware of these limitations, they may contribute to an unproductive learning environment for students. However, teachers who know and understand them may exploit these limitations to provide genuine learning opportunities and discussion points for their students. As students confront seemingly confounding situations on the graphing calculator, they are challenged to reconsider mathematical concepts from new points of view.

As an example, we exploit rounding error on the popular TI-84+ graphing calculator to encourage students to reconsider repeated decimals, limits, and equivalent representations of the number 1. The following vignette describes a typical teaching episode involving a preservice teacher, Ian Edwards, a classroom set of TI-84+ calculators, and a group of 8th grade algebra students.

"Okay, class! I want you to consider the number 0.9 for a moment. Go ahead and type the value into your calculator, then press ENTER." The class types the value into the TI-84 homescreen, as illustrated in Figure 4. Ian does the same on his calculator.

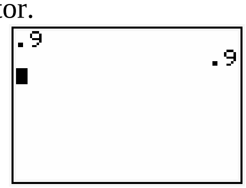


Figure 4: Results generated by entering nine-tenths into the TI-84+ homescreen.

"Okay. I have a question for you" Ian smiles. "How does the value you just typed in the calculator compare to 1? Is it larger than 1? Smaller than 1? Or the same?" Scratching his chin quizzically for a few moments, Ian waits as the hands of several students pop up in the air. Ian gestures to a student in the back of the room. "Kindly tell us about 0.9."

"Well. It's less than 1. Ten-tenths is 1, and that's only nine-tenths, so it's less," remarked the student.

"I think of it like 90 cents!" another student blurted out.

After a short discussion, it was apparent that students understood *why* 0.9 was less than 1. At this point, Ian moved the conversation along with the help of the calculator.

"Okay, now enter the numbers 0.99 and 0.999 into the calculator. Are these numbers less than 1?" The students successfully completed the task. In a similar manner, students agreed that 0.99 and 0.999 were less than 1, but they began to grow impatient with entering numbers into the calculator.

"What are you gettin' at, Mr. Edwards?" asked a student in the front row of the class. "This is baby stuff. This is too easy!"

"Okay. Okay. You all have indicated that 0.9, 0.99, and 0.999 are less than 1, and that the numbers are getting bigger each time, right?" Ian commented. "I made an interesting discovery last night. Type in the number 0.99999999999. It has eleven 9's in it. Is this less than 1?" Somewhat begrudgingly, students typed in the value into their calculators, however, this time the calculator output (as shown in Figure 5) surprised them.

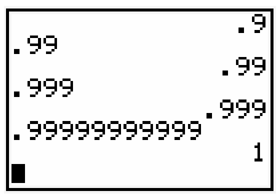


Figure 5: Unexpected output is generated when students type 0.99999999999 into the TI-84+ homescreen.

"Hey, it's 1!" several students quipped.
"No way!" remarked another.

The class was almost evenly divided into two camps. One camp believed that 0.99999999999 equaled 1. A student from this camp noted that "it's 1 'cause the calculator says so." The other camp believed that the value was really less than 1, but somehow the calculator didn't indicate this. What ensued was a lively discussion regarding the accuracy of the calculator, repeating decimal places, rational numbers, and the importance of critically analyzing text. "Don't believe everything you read!" noted Mr. Edwards to the class.

As homework, students were asked to convert decimal numbers from "decimal form" to "fraction form" (including the value of $0.\overline{9}$ which, in fact, actually equals 1). The introductory calculator activity proved useful to Mr. Edwards for several reasons. First, analysis of the calculator text encouraged students to think more deeply about repeating decimals and rational numbers. The activity "set the stage" for homework that followed. Secondly, it encouraged them to experiment with values on their calculators. Thirdly, and arguably most important, the activity encouraged students to view calculator text as *participant* as they challenged the results depicted on the screens of their calculators.

Calculator as Step-by-Step Solver
Earlier in this paper, we described the disappointment that one teacher felt as she observed a student solving the equation $2x + 8 = 14$ with the calculator's `SOLVE` command. As Figure 6 indicates, this is certainly possible.

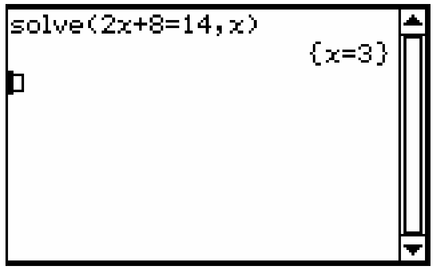


Figure 6: Students may use the `SOLVE` command to by-pass the symbolic manipulation (or mental mathematics) required to solve equations.

However, graphing calculators with symbolic manipulation capabilities (e.g., *TI-Nspire CAS*, *TI-89*, Casio *Classpad 300*) also support student step-by-step equation solving. Figures 7 through 9 highlight such features with a *Classpad 300*.

First the user types an equation to solve on the graphing calculator and presses EXE. The result of entering the expression is shown on the bottom right.

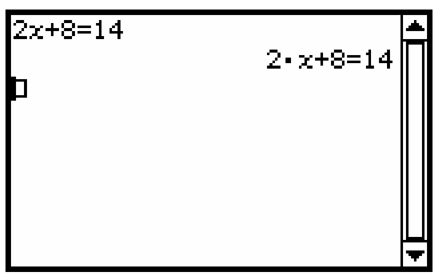


Figure 7: Entering an equation to solve on the *Classpad 300*.

The calculator treats algebraic equations as objects that may be manipulated symbolically. Conveniently, the *Classpad* stores results of its most recent calculation as `ans`. With `ans` equal to $2x + 8 = 14$, an operation such as `ans` - 8 subtracts 8 from each side of the equation.

As illustrated in Figure 8, the *Classpad* returns the equation $2x = 6$ when 8 is subtracted from each side of the original equation.

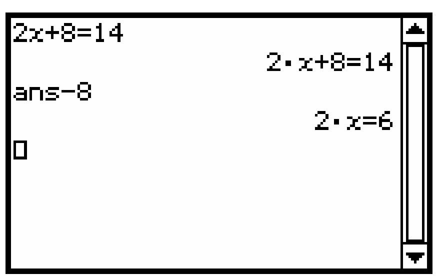


Figure 8: Using the *Classpad 300* to promote a transformation-based view of equation solving

Many students mistakenly subtract 2 from each side of the equation $2x = 6$ to “get x by itself.” Unfortunately, the result of such an operation does not *cancel out* multiplication by 2. This is shown in

Figure 9 (left). Using the calculator delete key, students can clear off incorrect steps and try again. This is suggested in Figure 9 (right).

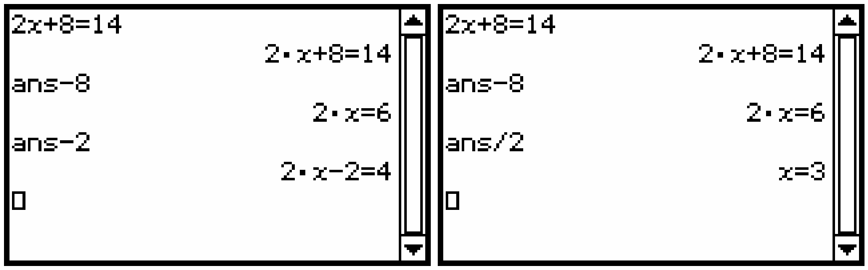


Figure 9: (Left) Using the TI-89 to highlight an error common among novice equation solvers; (Right) Solving the linear equation successfully.

Such techniques illustrate the use of graphing calculators as pedagogical tools, assisting students in constructing conceptual understandings that underlie meaningful symbolic manipulation. *Used responsibly, graphing calculators may encourage students to build stronger conceptual understandings of procedures that underlie symbolic manipulation.* In the preceding example, students interacted with calculator text as a *participant* as they learned to solve linear equations.

In Conclusion

We posit that a participatory view of the graphing calculator enables teachers and students to more fully realize technology's potential to strengthen student conceptual understanding of mathematics. The terms *object* and *participant* provide teachers with a language for discussing - and reflecting upon - the role of calculators in their teaching practice. Unaware of the calculator's capacity to foster experimentation and enhanced understanding of mathematics, teachers and students inadvertently use the devices as powerful *answer generators* without harnessing their power as pedagogical learning tools.

To better address the learning needs of our students, teachers need to recognize the importance of treating text as *participant* rather than *object*. This is particularly true of mathematics teachers as they explore calculator-based text with their students. The examples provided in the preceding paragraphs highlight classroom situations in which teachers and students considered the textual output of graphing calculators as participatory text rather than objectified text. In the examples, students were invited to envision calculator text as just *one view* of truth - often incomplete, flawed, and subject to interpretation. Students interacted with the calculator text as they questioned, interpreted, and connected it to prior mathematical knowledge - building stronger conceptual understandings of various topics in the process.

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